A Model of Multi-Store

A Model of Multi-Store Shoppers’ Buying Decisions

Abstract

We propose an analytical model of multi-store shoppers buying items from their shopping lists; specifically, “common items” that are available at competing stores. Multi-store shoppers buy some common items at the first store they visit, others are deferred to a competing store. These buying decisions depend on the prices observed at the first store and uncertainty about savings if purchases are deferred to a competing store. Analysis of our model shows that, if multi-store shoppers enjoy psychological benefits (in addition to rational

1. Introduction

Consider a grocery shopper's purchase decisions. She goes shopping to buy certain items, which are usually recorded on a shopping list (Spiggle 1987, see Kahn and McAlister 1997, pp.118-9, for a discussion of shopping lists). Given that shopping list, store choice models assume that the shopper visits whichever store minimizes her total cost of shopping; i.e., the cost of travel and multi-store shoppers do not buy at the low price more often, given that most prior research assumes that the objective of multi-store shopping is to search for deals.

- Second, our empirical data (again, see §4.1 for details) show that if the first store that a shopper visits offers a price that is less than or equal to the second store's price, then the shopper buys at that low price 76.9% of the time. If, on the other hand, the first store that a shopper visits does not offer the low price (i.e., its price is higher than the price offered at the second store) then the shopper buys at the low price only 55.8% of the time.[2] This analysis suggests that the order in which multi-store shoppers visit stores affects the implication is that uncertainty about prices at the second store affects multidiscounting independent of competitor

primary grocery store, increasingly buying food and packaged goods at other stores and retail formats.

Three existing literature streams are particularly relevant to multi-store shopping: marketing researchers have studied cherry-picking, or shopping for bargains across stores; economists have considered models of sequential search for grocery products; and social psychologists have discussed the psychological (i.e., non-economic) benefits of saving money.

2.1. Cherry-Picking

A number of recent studies have investigated cherry-picking--shopping for bargains across stores--focusing on who cherry-picks, how much they cherry-pick and how retailers' pricing and store location decisions affect cherry-picking behavior. Note that the term "cherry-picking" implies that the motivation for multi-store shopping is to buy at a lower price; as we will explain in §3, our model of multi-store shopping allows for other motivations as well.

Cherry-picking has been used in game theoretic models of the retailer/shopper interaction. For example, Lal and Rao (1997) developed a model that segments shoppers into those who are time constrained and those who cherry-pick. Dreze (1999) analyzed a segment of shoppers who are price sensitive with low travel costs and so can be induced to cherry-pick by retailer price deals. Both studies argue that cherry-pickers will travel to multiple stores to take advantage of price deals because of their low opportunity cost of time. Moreover, cherry-picking shoppers are generally assumed to be less profitable than other shoppers for retailers.

Other cherry-picking studies have taken a more empirical approach. Fox and Hoch (2005) found that cherry-picking is materially important for retailers, on average, and iii) is less likely to secure bargains. As a result, the primary store is not adversely affected by cherry-picking as much as the *secondary* store. Fox and Hoch also found evidence that cherry-picking trips are planned, with more than twice as much spent on such trips compared to single-store shopping trips. In terms of demographics, cherry-picking behavior was found to be positively associated with household size, home ownership and senior citizenship, but negatively associated with working adult females and income. The authors also found that, depending upon the shopper's wage rate, cherry-picking is economically rational behavior for a substantial proportion of households.

Talukdar, et al. (2010) investigated the profit impact of extreme cherry-picking on retailers, where extreme cherry-pickers were defined as those who generated a negative profit contribution at their secondary store--

search skills, finding that both are positively related to the two dimensions of price search (including cherry-picking) and to shoppers' savings from price search. These mavenism and perceived search skill findings suggest that psychological factors may affect search behavior and consequently support the inclusion of psychological benefits in our model of multi-store shopping behavior.

2.2. Grocery Store Price Search

In economics, there have been several studies of the search for information about frequently purchased goods across grocery stores (e.g., Stigler 1961). These studies assumed sequential se

offers attractive assortments in categories of interest, then the customer might choose to visit the second store regardless of potential savings on common items. In this case, all travel costs would be treated as sunk costs, so the fixed cost associated with purchasing psychological benefits from saving money; thus, the deferment decision is not influenced by the probability of realizing a savings. The shopper maximizes expected savings (across all items on the list), taking into account the fixed cost of a visit to Retailer 2

$$\underset{\lambda_t}{Max}\ \ E\left\{\lambda_t^T Q_t d_t\right\} ! \ k \, , \qquad (1)$$

where d_t is a vector of price differences, , Q_t is a diagonal matrix of required purchase quantities, $_t$ = !"#$($_{1t}$! $_{!t}$! ! ! $_{!"}$), and *E* is the expectation operator. Observe that positive values in the price difference vector imply a positive contribution to savings by deferring purchase to Retailer 2. The shopper's maximization of expected savings in (1) implicitly assigns $!_{it}$ = 1 for each product where E(d_t) > 0 and $!_{it}$ = 0 otherwise. Because multi-store shoppers visit both retailers by definition, the optimality of this decisiodein,)purcs de (dQ q0.24 0 0 C these decision weight functions are nonnegative and strictly monotone increasing. These assumptions ensure that, all things being equal, i) the shopper is more likely to defer item purchases if the expected savings of doing so is greater, holding the probability of realizing a savings fixed; and ii) the shopper is more likely to defer item purchases if the probability of realizing a savings (by deferring) is greater, holding the expected savings fixed. The mathematical program for this decision model in period t is

$$\underset{\lambda_t}{Max}\ g\ E\left\{\lambda_t^T Q_t d_t\right\} ! \ k\ + h\ \Pr\left\{\lambda_t^T Q_t d_t > k\right\}\ , \tag{2}$$

where $E\left\{\lambda_t^T Q_t d_t\right\} ! \ k$ is the *expected savings* from deferring a proportion of purchases to Retailer 2 and Pr λ_t^{T}

where δ_t is the vector of expected price differences

2. $\lambda_{it}^{*} > 0$ for $\in$ $^{+}$

3. For any $i, j \in I_t^{+}$ with $\lambda_{it}^{*} < 1, \lambda_{jt}^{*} < 1$, $\dfrac{\lambda_{it}^{*}}{\lambda_{jt}^{*}} = \dfrac{\delta_{it}}{\delta_{jt}} \cdot \dfrac{\sigma_{jt}^{2}}{\sigma_{it}^{2}} \cdot \dfrac{q_{jt}}{q_{it}} = \dfrac{\delta_{it} / q_{it}\sigma_{it}^{2}}{\delta_{jt} / q_{jt}\sigma_{jt}^{2}}$.

4. If λ_{it} λ_{jt}, then .

We provide a proof of these conditions in Appendix A. Observe that these conditions do not depend on the fixed cost *k*, a construct of the model which is neither measured nor calculated.

Parts 1 and 2 of the theorem tell us that every item whose expected price is lower at Retailer 2, , should be purchased in some proportion at Retailer 2, while every item whose expected price is *not* lower at Retailer 2, $\notin$ $^{+}$, should not be purchased at Retailer 2. Part 3 of the theorem defines the relationship between any two products that should be bought in some proportion at both retailers. Part 4 of the theorem specifies the relationship that must hold if the shopper prefers purchasing item *i* in greater proportion than item *j* at Retailer 2. The necessary condition is that the ratio $\frac{!}{}$ for item *i* must exceed the same ratio for item *j*. This is an intuitively appealing condition since it incorporates both expected savings and price uncertainty in a simple and parsimonious way. Moreover, it confirms the simple intuition that the shopper should defer those purchases to the second retailer that have the greatest certainty of contributing to savings. To our knowledge, no existing models of decision-making under price uncertainty use this ratio. We note that it is similar to the Sharpe ratio used in financial portfolio theory, except that the denominator in our expression uses the variance, instead of the standard deviation, and includes a quantity scale factor. The quantity scale factor arises because buying larger quantities increases the expected contribution to savings but also, to a greater extent, the uncertainty of the contribution to savings by purchasing at Retailer 2.

Proposition 2 – For shoppers, the decision to function as shown in Figure 1 and therefore estimable using a binary choice function such as logit or probit, both of which are appropriate and well supported for empirical applications visiting the second retailer are not necessarily symmetric. In Figure 2, the additional setup cost of going from Retailer A to Retailer B (the route shown in bold) is greater than the additional setup cost of going from Retailer B to Retailer A (the dotted route). Retailer A is nearly "on the way home" after visiting Retailer B whereas Retailer B is "out of the way" after visiting Retailer A. Finally, note that the routing decision is dependent on the distributions of prices offered by the two retailers. The relationship between retailer pricing and the shopper's optimal routing is examined in detail in Bhaskaran and Semple (2012). This study shows that differences in the skewness of retailers' price distributions alone can materially affect a shopper's expected purchase costs, resulting in different optimal routes.

Place Figure 2 about here

4. Empirical Demonstration

In this section, we use actual common item purchases made by multi-store shoppers to demonstrate that some shoppers enjoy psychological benefits, in addition to the economic benefits, of saving money. Note that our objective is to provide a demonstration of the analytically-derived propositions in §3, not to conduct a generalizable test of shopping behavior.

4.1. Data

We use IRI panel data from the Chicago market over 104 weeks between October 1995 and October 1997. Panelists recorded the UPCs (uniform product codes) of all packaged goods products purchased on all trips to a wide variety of retailers using in-home scanning equipment, identifying the retailer by store chain rather than by individual store. Developing panelists in the dataset

households in our final dataset, we have been intentionally conservative in identifying multi-store shopping purchases that reflect the assumptions of our model. As mentioned above, limiting our dataset to experienced multi-store shoppers ensures that their shopping trips were planned in advance (Fox and Hoch 2005). Selecting common item purchases only if both stores were visited without intervening consumption ensures that the shopping list was not increased between visits. As a result of this conservative approach, the 873 purchases in the dataset are, to the extent possible, representative of our analytical model. And while our intention is simply to demonstrate that some multi-store shoppers enjoy psychological benefits, the large number of observations per household supports the reliability of our findings.

included in our dataset are presented for comparison, and it appears that they shop considerably less than the multi-store shoppers in our dataset. We observe that the frequent multi-store shoppers in our dataset made 187.25 (= 43.80 + 143.45) total store visits while the other shoppers made only 64.02 (= 2.40 + 61.62) total store visits--nearly three times fewer.

Place Table 2 about here

4.2. Variable Definitions

We carry forward the notation from the analytical model; however, our empirical analysis is conducted at the individual level so we will add a subscript for household. Accordingly, the dependent variable in our econometric model is the probability % that household *h* purchases common item *i* on trip *t* at decision to defer purchase to Retailer 2 is an increasing function of the contribution to expected savings, divided by the product of the quantity and the contribution to variance of savings. For household *h* purchasing item *i*, the predictor is $\frac{!}{tl}$. The two quantities u and $\frac{!}{tl}$ will play a critical role in our empirical analysis.

We assume that the shopper's information about price savings comes from previous multi-store trips on which the household made purchases in the category. During those trips, the shopper would have access to comparative pricing information for common items in the category. Because they depend on shopping history, contributions to expectation and variance of savings for each common item are household-specific, hence the *h* subscript. Moreover, because the shopper observes the prices of common items at Retailer 1 before making deferral decisions, we assume that she uses this information to condition contributions to expected savings (for [illegible] where is an indicator variable which takes the value 1 if item

4.3.1. Model Forms. The decision weights $!_E$ and $!_P$ play a critical role in our analysis. Consider the following possible scenarios:

- $!_E!0$ but $!_P$=0; in this case, all shoppers are motivated by the economic benefits, but not psychological benefits, of the expected saving money--the probability of deferring purchase to the second retailer will increase as the contribution to expected savings increases.
- $!_P!0$ but $!_E$=0; in this case, all shoppers are motivated by both the psychological and economic benefits of the saving money--the probability of deferring purchase to the second retailer will increase as the ratio of contribution to expected savings divided by contribution to variance of savings increases.

These scenarios raise the possibility that shoppers may differ in their motivations, the way they process information, and the way they make decisions. Accordingly, we will allow for where the decision weights and category-specific store loyalty parameters now vary by segment. However, as stated above, we will never allow both $!_E$ and $!_P$ to be non-zero within a given segment. Rather, we align each segment with a specific motivation by setting one of these parameters to zero.

4.3.2. Estimation. We model the probability that household *h* chooses to buy common item *i* from her shopping list on trip *t* at Retailer 1, the first retailer visited on that multi-store shopping trip, as

$$_{hit} = \frac{e^{U_{hit}}}{+ e^{U_{hit}}} \quad . \tag{12}$$

As discussed in §3.2, because the purchase decision function is approximately piecewise-linear, the logit is an appropriate model form for estimating the probability (for a detailed discussion, see Ben-Akiva and Lerman 1985, pp. 67-72).

In general, the unconditional likelihood for a multi-store shopper with common item purchase vector y_h can be written as

(13)

where $L(y_h|\#)$ is the conditional likelihood with parameters # (=", $!_E$, and $!_P$), and $F(\bullet)$ is the mixing distribution. It can be shown that a continuous mixing distribution function $F(\bullet)$ can be consistently estimated with a finite number of *S* mass points (cf. Simon 1976), i.e.,

$$= \ \sum_{=1} \tag{14}$$

where $\#_s$ is the vector of parameters, and $\$_s$ is the mixing proportion or segment share for segment *s*, such that 0"$\$_s$"1 and $\$_1 + \$_2 + \ldots. + \$_S$=1. Parameters are estimated using both the EM algorithm and the Newton-Raphson method. To decrease the chance of local maxima solutions, we use multiple sets of random start values. Within each set of random start values, we perform a number of iterations and continue with the best solution until convergence.

4.4. Empirical Results

Table 3 provides a description of alternative model forms along with goodness of fit statistics and hit rates. Table 4 provides parameter estimates for the best fitting model. We use AIC3 (Andrews and Currim 2003) and BIC (Schwartz 1978) to compare model fits.[12]

Table 3 is divided into two sections. The first section considers model forms in which all

Because the price savings that a shopper may realize by deferring common item purchases is effectively a risky return, we estimate an alternative model for this decision which includes an additive risk term. Specifically, we assume that multi-store shoppers' decisions to defer purchase of common items to Retailer 2 depend.2 (vi) 0.2wnifor were included. We selected this sample to test whether our empirical results depend on households' multi-store shopping expertise.

Robustness Sample B: This sample reflects a different assumption about how shoppers develop expectations about items' contribution to price savings. Adopting a *rational expectations* approach, this sample incorporates the entire history of price differences between retailers, regardless of which prices the household might have observed. The implication is that multi-store shoppers' common item purchase decisions are made *as if* the shoppers know the entire history of price differences for common items.

In the case of both robustness samples, we fit all model forms described in Table 3. For both robustness samples, *M*3_1 was again the best-fitting model in terms of both goodness-of-fit and hit rate. Interestingly, some patterns in the parameter estimates are noteworthy. For *Robustness Sample A*, which imposed a stricter multi-store shopping experience sar mmmms.2 (r.4 (m) 02 (e) 0.2 (t)) .2 (r27.m) 0.2.2 ( ) -2 (c) 0..4 (m) 09.4 (a) -176.6 (m) 2 (pos unbiased) over contribution to variance in savings (which has a downward bias) is overstated, with some highly influential observations due to the small denominator. This explains the weaker relationships found in *Robustness Sample B.*

5. Discussion, Limitations and Future Research

We now return to the multi-

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Appendix A

Proof of Theorem 1. Because 0, consider the solution $\lambda_{it} = 1$ for all $\in {}^{+}$, and $\lambda_{it} = 0$ for $\notin {}^{+}$. $\sum_{\in} \lambda \quad \delta \quad - \quad 0$ for this solution. Observe that a solution with positive expected savings dominates all solutions with non-positive expected savings, i.e., both g and h are larger with positive expected savings. We may therefore assume that $\sum_{\in} \lambda^{*} \quad \delta \quad - \quad 0$ in any optimal solution vector λ_t.

Now we show that $\lambda_{it}^{*} = 0$ for all $\notin {}^{+}$. Suppose this were not the case, and $\lambda_{it}^{*} > 0$ for some $\notin {}^{+}$. Then construct a new vector $\lambda_t^{'}$ as follows: $\lambda_{jt}^{'} = \lambda_{jt}^{*}$ for $j \,!\, i$, $\lambda_{jt}^{'} = 0$ for $j = i$. Observe (a) $\lambda_t^{'T} Q_t \delta_t > \lambda_t^{*T} Q_t \delta_t$, (b) $k - \lambda_t^{'T} Q_t \delta_t < k - \lambda_t^{*T} Q_t \delta_t < 0$, and (c) $!_t^{'T} Q_t \;\; {}_t Q_t \, !_t^{'} < !_t^{*T} Q_t \;\; {}_t Q_t \, !_t^{*}$. Observation (a) implies g will increase for the new solution $\lambda_t^{'}$. Observations (b) and (c) imply the argument of Φ will decrease, and so h will increase. This contradicts the optimality of λ_t and implies $\lambda_{it}^{*} = 0$ for all $\notin {}^{+}$. This proves part 1 of the theorem.

We now show that $\lambda_{it}^{*} > 0$ for *all* $\in {}^{+}$. Because $\sum_{\in} \lambda^{*} \quad \delta \quad - \quad 0$, at least one component of the optimal solution is positive. Let that component be $\lambda_{it}^{*} > 0 \quad \in {}^{+}$. Now suppose for some (other) item $\in {}^{+}$

Observe that h's argument is $1-\Phi\left(\frac{k-\lambda_t^T Q_t \delta_t}{\sqrt{\lambda_t^T Q_t \Sigma_t Q_t \lambda_t}}\right)$. The term $k \# \lambda_t^T Q_t \,!_t$ in the numerator of Φ's argument is identical for λ' and λ (it's the negative of expected savings). However, the denominator of Φ's argument does change. In fact, straightforward algebra reveals the expression under the root changes by a net amount of

.

to preserve optimality. Dividing this expression by

Appendix B

Numerical Study.

μ_h = 0. The diagonal matrix of quantities Q simply scales the expected savings and variance of savings terms. It is set to the identity matrix.

In conducting our numerical study, we systematically varied $(_g$, $(_h$, $)_g$ and $)_h$. For each combination of slopes and exponents, we assumed a shopping list of fifty items. The

- for all values of the ratio75 (or a) 0.2 (l) 0.2 (l) 0.2 (vanov(i) 0.2 (75 (or a c 0.2 (l(i) 0.2 r (t) 0.275 (or ai) ·

Appendix C

The conditional contribution to expected price savings is computed as follows:

, (C.1)

where

is the unconditional contribution to expected savings,

is the expected price at Retailer 1,

is the covariance between the price at Retailer 1 and the contribution to savings from deferring to Retailer 2, and

is the variance of prices at Retailer 1.

Similarly, the conditional contribution to variance of price savings is computed as follows

, (C.2)

where is the unconditional contribution to variance of savings. For our dataset, the conditional contributions to expectation and variance of savings were computed iteratively

Exhibit 1

Summary of Notation	
Symbol	*Meaning*
	Item-level stochastic prices for Retailer 1 on trip *t*
	Item-level stochastic prices for Retailer 2 on trip *t*
	Proportion of item purchases at Retailer 2 on trip *t*
	Diagonal matrix of required quantities on trip

Figure 1
The Optimal Proportion of an Item to Buy at the Second Retailer

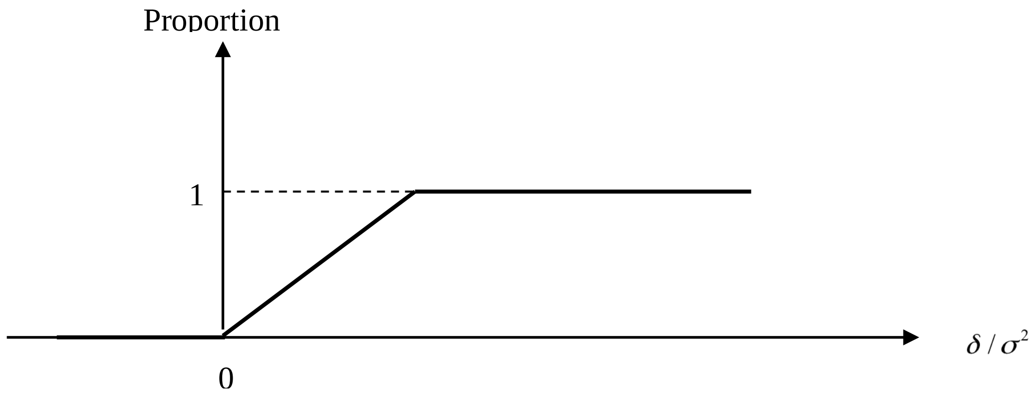

Figure 2
Routes of Return Beginning with Different Retailers

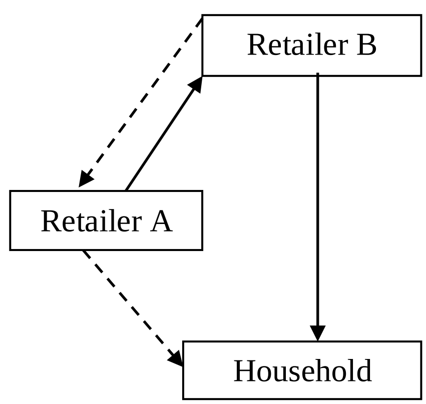

Table 1
Panelist Demographics

	Multi-Store Shoppers Included in Dataset (n=51)		Other Shoppers Not in Dataset (n=485)	
	mean	std dev	mean	std dev
Family Size	3.25	1.44	2.87	1.44
Household Income (x$1,000)	53.4	25.9	51.3	26.3
Working Adult Female	0.588	0.497	0.640	0.480
College Education	0.216	0.415	0.202	0.402
Home Owner	0.922			

Table 4
Model _1 Parameter Estimates
